CE 672 – Numerical Methods in Structural Mechanics

Assignment No. 1

1. Write down each of the following expressions using summation convention:

   a) $g_{21}g_{11} + g_{22}g_{21} + g_{23}g_{31}$
   b) $a_1 x_1 x_3 + a_2 x_2 x_3 + a_3 x_3 x_3$.

2. If $f = a_{ij} x_i x_j$, show that it is always possible to write $f = b_{ij} x_i x_j$; where b_{ij} are components of a second-order symmetric tensor.

3. Prove that if a_{ij} are the components of a second-order skew-symmetric tensor, then $a_{ij} x_i x_j = 0$.

4. Show that the tensor whose components are $T_{ij} = e_{ijk} a_k$ is skew symmetric.

5. Show that the determinant $\det\left|A_{ij}\right|$ may be expressed in the form $e_{ijk} A_{1i} A_{2j} A_{3k}$.

6. Evaluate:

   a) $d_{ij}\, d_{ik}$
   b) $d_{ij}\, d_{ik}\, d_{jk}$
   c) $e_{ijk} e_{kij}$
   d) $e_{ijk} a_j a_k$

7. Establish the identity:

$$e_{pqs} e_{mnr} = \begin{vmatrix} d_{mp} & d_{mq} & d_{ms} \\ d_{np} & d_{nq} & d_{ns} \\ d_{rp} & d_{rq} & d_{rs} \end{vmatrix}$$

8. Use index notation to prove the vector identities:

   a) $\vec{\nabla} \times \vec{\nabla} f = 0$
   b) $\vec{\nabla} \cdot \vec{\nabla} \times \vec{a} = 0$

9. Find the principal values and principal directions of the second-order symmetric tensor ${}^2\vec{T} = T_{ij} \vec{i}_i \vec{i}_j$ whose components are given by:

$$T_{ij} = \begin{pmatrix} 7 & 3 & 0 \\ 3 & 7 & 4 \\ 0 & 4 & 7 \end{pmatrix}$$

10. Show that a second-order skew symmetric tensor ${}^{2}\vec{b} = b_{ij}\vec{i}_i\vec{i}_j, b_{ij} = -b_{ji}$, has only one principal direction.